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1/f noise, disorder and dimensionality

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Résumé. — Nous discutons le rôle joué par le désordre et la dimensionnalité critique inférieure dans l'existence d'un spectre en $1/f$. Nous présentons les résultats d'une simulation numérique d'un verre de spin d'Ising bidimensionnel, montrant un spectre $1/f$ dans un domaine très étendu des températures et des fréquences.

Abstract. — We discuss the rôle of disorder and of the lower critical dimension in producing a $1/f$ spectrum. We present a numerical simulation for a bidimensional Ising spin glass, showing a $1/f$ spectrum in a remarkable range of temperatures and frequencies.

1. Introduction.

The so-called $1/f$ noise (or flicker noise) deserves careful investigations : it is found in a wide class of physical phenomena [1, 2], apparently completely uncorrelated, and it is very difficult to justify from a mathematical point of view [3]. The systems we are considering exhibit, in some physical observable, a fluctuation $\varphi(t)$, whose power spectrum

$$S(\omega) = \frac{1}{T} \left| \int_0^T dt e^{i\omega t} \varphi(t) \right|^2, \quad (1.1)$$

for low values of the frequency ω , is found to behave as

$$S(\omega) \sim \frac{1}{\omega}. \quad (1.2)$$

Originally $1/f$ noise was found in the voltage fluctuations across a conductor carrying a constant electric current : moreover it exists in the fluctuations of oceanic currents, of sunspot number, of the luminosity of some stars, and the loudness of classical and

modern music, etc. In the following we will just examine the problem in a solid state physics context (resistors with electric noise) : it should be noticed however that the overall picture we are proposing (quenched disorder and self-similarity) can be eventually used as a general mainframe toward the understanding of the various $1/f$ effects.

Since the flicker noise was observed in a very large class of resistors [4] (metallic films, carbon resistor, many kinds of diodes and junctions) it was suggested that a universal mechanism is needed to explain this feature.

Therefore some attempts have been made to explain the $1/f$ noise in terms of either turbulence or critical phenomena. Indeed both these phenomena exhibit self similarity and power law behaviour in the spectra of the correlation functions. An explanation of the flicker noise in terms of fully developed turbulence has been given by Handel, Teitler and Osborne [5, 6] who considered the voltage fluctuations as caused by velocity fluctuations of the electronic flow and used perturbative techniques (quasi-normal approximation) and similarity hypothesis.

This approach assumes that the $1/f$ noise is a non-equilibrium phenomenon, while the experimental results seem to clearly indicate that it is an equilibrium phenomenon : moreover the rôle of dimensionality and defects is neglected, and the $1/f$ behaviour turns out to be independent of the spatial dimension. On the contrary it is well known that the $1/f$ noise is not observable in the usual metallic 3-dimensional resistors and is exhibited by 2-dimensional systems (metallic films as well as carbon resistors which have fractal dimension close to 2) and by « dirty » systems (interaction with surfaces, absorbed systems). So the analogy between turbulence and flicker noise does not seem very deep.

As far as the critical phenomena are concerned, the universality due to the divergence of a characteristic length scale and the power laws of correlation functions in a 2nd order phase transition occurs only in a narrow range around the critical temperature. However, the self-similarity structure of the $1/f$ noise stays up to very long time scale in a large temperature range (~ 100 K).

Detailed descriptions of flicker noise in different physical systems probably require different explanations : we think however that the main mechanisms governing what is happening in these systems could have in common many fundamental features, and that they have to be emphasized. Our point of view is that the self-similar structure of flicker noise should be connected to the presence of defects and random interaction, and that dimensionality could play an important rôle. Recently results in this sense have been obtained : it has been showed that systems with fractal dimension $D_F \sim 2$ [7], as well as random walks in random potentials obeying some scaling laws [3], exhibit a $1/f$ spectrum. Here we will elaborate further on the rôle of disorder (section 3) and of the lower critical dimension (section 4), presenting numerical simulations, on a bidimensional Ising spin glass, that will support our thesis.

2. General considerations.

We will consider a statistical system S , and its dynamical evolution, governed by the Hamiltonian H . We will assume S being at equilibrium, (or possibly at quasi-equilibrium) and we will indicate by $A(t)$ a generic observable. The power spectrum $P_A(\omega)$ of $A(t)$ is defined by

$$P_A(\omega) = \lim_{T \rightarrow \infty} \frac{1}{T} |A_T(\omega)|^2 = \int_{-\infty}^{+\infty} dt C_A(t) e^{i\omega t}, \quad (2.1)$$

where

$$A_T(\omega) = \int_{-T/2}^{+T/2} dt A(t) e^{i\omega t}, \quad (2.2)$$

and $C_A(t)$ is the time correlation function

$$C_A(t) = \langle A(t) A(0) \rangle. \quad (2.3)$$

By $\langle \cdot \rangle$ we indicate a statistical average. In deriving the second identity of relation (2.1) we have used the invariance under time translations :

$$\langle A(t + t') A(t') \rangle = \langle A(t) A(0) \rangle. \quad (2.4)$$

Let us neglect the possibility of systematical time oscillations of the observable $A(t)$: then if the power spectrum shows a $1/f$ noise, i.e.

$$P_A(\omega) \sim \frac{1}{\omega} \quad (2.5)$$

this implies

$$C_A(t) \sim \ln t. \quad (2.6)$$

We define

$$\bar{A}_T = \frac{1}{T} \int_{-T/2}^{+T/2} A(t) dt \quad (2.7)$$

and we find that condition (2.6) implies

$$\langle \bar{A}_T^2 \rangle = \frac{1}{T^2} \left\langle \int_{-T/2}^{+T/2} dt \int_{-T/2}^{+T/2} dt' A(t) A(t') \right\rangle$$

using (2.4)

$$= \frac{1}{T} \int_0^T dt \ln t \simeq \ln T$$

for large T . So

$$\lim_{T \rightarrow \infty} \langle \bar{A}_T^2 \rangle = \infty. \quad (2.8)$$

A logarithmic divergence is very weak : it will be completely ineffective if the coefficient of the diverging logarithm is small on the scale of the observation times. We can conclude that also in a situation in which $\langle \bar{A}_T^2 \rangle$ is bounded *a priori*, a $1/f$ behaviour of the power spectrum can be observed in a large region of frequencies.

We could try to consider a time dependent Hamiltonian H' , by adding to H the operator $A(t)$ times a small parameter $\varepsilon(t)$

$$H'(t) = H + \varepsilon(t) A(t). \quad (2.9)$$

We define the response function in the usual way

$$R_A(t) = \frac{\delta A(t)}{\delta \varepsilon(t)}, \quad (2.10)$$

and we get (fluctuation-dissipation theorem)

$$R_A(t) = \beta \dot{C}_A(t). \quad (2.11)$$

In presense of $1/f$ noise (2.6) holds and

$$R_A(t) \sim \frac{1}{t},$$

i.e. the response function in Fourier space

$$R_A(\omega) \sim \ln \omega.$$

Unluckily this procedure is not practical for semi-conductors, since it is very problematic to add to the time independent Hamiltonian a resistivity term. However the argument shows that all systems whose response function has a logarithmic dependence on frequency have to exhibit 1/f noise.

Typical systems in which there is a strong experimental evidence that the response function depends on the frequency on a logarithmic scale are provided by real glasses and spin glasses [8].

Then we expect to observe a 1/f behaviour of $P_A(\omega)$ if we identify $A(t)$ with the total magnetization (the energy in the canonical formalism) for a spin (real) glass.

These considerations are of kinematical and/or thermodynamic nature. Actually, at least as far as spin glasses are concerned, there is a general agreement that fluctuations on a very large scale are generated by the hopping of the system among different equilibrium or quasi-equilibrium states [9, 10] ⁽¹⁾.

A logarithmic frequency dependence of the response functions may also be found in systems at the lower critical dimension : i.e. those systems for which a possible transition is forbidden by the appearance of logarithmic divergences.

In the following we shall discuss in detail these two mechanisms inducing 1/f noise : disorder, in section 3, and lower critical dimension in section 4.

3. Spin glasses.

We will consider an Ising spin glass in two dimensions, with the Hamiltonian

$$H = - \sum_{\langle ij \rangle} J_{ij} \sigma_i \sigma_j - h \sum_i \sigma_i, \quad (3.1)$$

where $\sigma_i = \pm 1$, the sum $\langle i, j \rangle$ runs over all the nearest neighbours sites on the lattice, and the couplings J_{ij} are random quenched variables which can assume the values ± 1 . They are uniformly distributed according to the probability

$$P(J_{ij}) = \frac{1}{2} \{ \delta(J_{ij} + J_0) + \delta(J_{ij} - J_0) \}. \quad (3.2)$$

h is a fixed external field. When not explicitly stated h will be set equal to zero.

The model defined by (3.1) and (3.2) was analysed in reference [11]. Its static susceptibility χ (the response function of the magnetization in our terminology) has a sharp peak at the freezing temperature T_f , which depends on the observation times.

Actually, the freezing of a group of spins into random directions is a non equilibrium, dynamic phenomenon, rather than an equilibrium phase tran-

sition, in 2 and possibly 3 dimensions [12]. In two dimensions T_f has to vanish at a unattainable equilibrium, but $k_B T_f \simeq 1.4 J_0$ [5] in the experimental or computer observation time.

We are thus interested in the time evolution of the system : we will simulate its dynamic by a Heat Bath [13] and by a Monte Carlo [14] algorithm.

Let us define the two dynamics we will use in our numerical simulations. At $h = 0$ the Boltzmann weight is

$$e^{-\beta H} = \exp \left\{ \beta \sum_{\langle ij \rangle} J_{ij} \sigma_i \sigma_j \right\}. \quad (3.3)$$

We want to consider a time evolution that will eventually lead the spins of our system to be equilibrated according to the probability measure (3.3). We will change one spin at once, for example

$$\sigma_k \rightarrow \sigma'_k. \quad (3.4)$$

The Hamiltonian (3.1) is local, and if we change the spin σ_k just the 2d terms will change, i.e.

$$\sigma_k = \sum_{m=1}^{2d} J_{km} \sigma_m, \quad (3.5)$$

where the sum runs over the first neighbours of k .

In the Heat Bath procedure we will consider the probability for σ'_k being +1 or -1, given its first neighbours in our given configuration :

$$P(\sigma'_k = +1) \equiv P(+)= \frac{1}{Z} e^{\sum_i J_{ki} \sigma_i}, \quad (3.6)$$

where

$$Z \equiv e^{\sum_i J_{ki} \sigma_i} + e^{-\sum_i J_{ki} \sigma_i}, \quad (3.7)$$

and

$$P(\sigma'_k = -1) \equiv P(-) = 1 - P(+). \quad (3.8)$$

It is clear that if we already have a Boltzmann distribution it will be transformed in itself by this procedure. We will extract a random number ρ , uniformly distributed in (0.1), and if $\rho < P(+)$ then $\sigma'_k = +1$. Otherwise $\sigma'_k = -1$. Since this procedure is ergodic it will eventually bring any starting configuration to Boltzmann equilibrium.

In the Monte Carlo scheme we will define a trial new spin

$$\sigma_k^T = - \sigma_k. \quad (3.9)$$

Now we will compute

$$\begin{aligned} \Delta H(\sigma_k \rightarrow \sigma_k^T) &= (\sigma_k^T - \sigma_k) \sum_i J_{ki} \sigma_i \\ &= -2 \sigma_k \sum_i J_{ki} \sigma_i. \end{aligned} \quad (3.10)$$

If $\Delta H < 0$ then $\sigma'_k = \sigma_k^T$. If $\Delta H > 0$ we compute

$$p \equiv e^{-\beta \Delta H}, \quad (3.11)$$

and extract ρ , uniformly distributed in (0.1). If $\rho > p$ then $\sigma'_k = \sigma_k^T$, otherwise $\sigma'_k = \sigma_k$.

⁽¹⁾ The mechanism recalls that of point C, section 3, of the second part of [3].

In both schemes all the lattice is updated, changing a spin at once. For the Heat Bath method the location of the spin to be changed was randomly chosen : for MC the lattice was updated sequentially. The results given by the two algorithms are completely equivalent. We set $J_0 = 1$, and $T = \beta^{-1}$.

Although for the two dynamics we are considering there is, to the best of our knowledge, no equivalent of the fluctuation-dissipation theorem, the large time behaviour is supposed to be qualitatively the same inside a large class of reasonable dynamics.

We define a « sweep » to be a complete update of the lattice in the sequential scheme and the updating of twice the number of sites contained in the lattice for the scheme in which sites are randomly chosen.

As stressed before we expect that the long time behaviour of the magnetization is characterized by its jumps across the barriers among valleys of metastable, quasi-equilibrium states in the phase space. In our simulations (done on a 16^2 lattice) we were starting from a random configuration of the spins, cooling it down from $T_0 \sim 2 T_f$ to the T of interest in 30 sweeps. The realization of the random couplings was quenched. Now we performed 2^{12} sweeps, recording the magnetization

$$m(t) \equiv \frac{1}{N^2} \sum_i \sigma_i(t) \quad (3.12)$$

at the end of any sweep, and eventually computing its Fourier transform

$$\tilde{m}(\omega) \equiv \int e^{i\omega t} m(t) dt. \quad (3.13)$$

We averaged the power spectrum (see (2.1))

$$S_m(\omega) \equiv |\tilde{m}(\omega)|^2 \quad (3.14)$$

over 50 different time evolutions in different realizations of the random couplings.

Our results are shown in figures 1-3 : $S_m(\omega)$ is found to behave as ω^{-1} for $T \in (0.7, 1.2)$, where we recall that the « freezing » temperature is ~ 1.4 on our time scale, and for frequencies up to $\omega_{\max}/2$. The $1/\omega$ signal is absolutely clean. Looking at the dependency of $S_m(\omega)$ over T we found

$$S_m(\omega) = \frac{C(T)}{\omega}, \quad (3.15)$$

where $C(T) \sim -0.6 + 1.3 T$, for $T \in (0.7, 1.2)$, see figure 4. It is interesting to note that it is experimentally known [11] that in some solid state systems, $C(T) \sim \text{const.} \times T$. Our results indicate that the linear dependence on T holds down to $T \sim 0.6$. On our observation time below this temperature the system is bounded in a valley, and its specific heat is very close to zero. For $T \gtrsim T_f$ the spectrum becomes white : the phase space landscape no longer has a self similar structure.

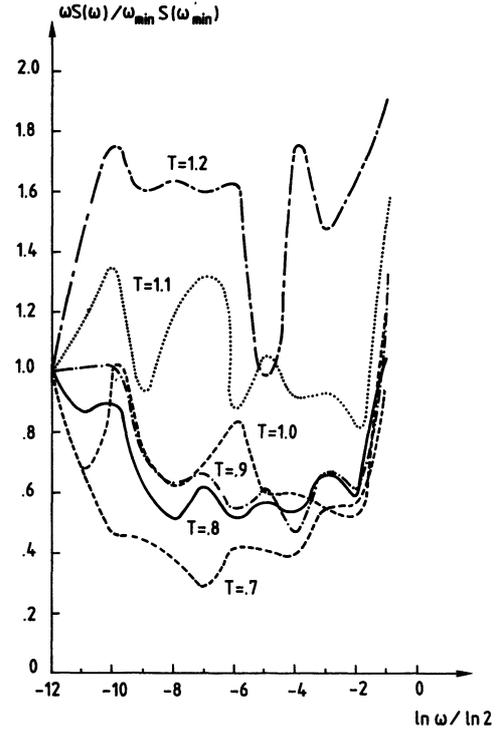


Fig. 1. — $S(\omega) \omega / S(\omega_{\min}) \omega_{\min}$ versus $\ln \omega / \ln 2$, at various temperatures (from 0.7 to 1.2). The curves are a guide for the eye. Heat Bath dynamics, 16^2 lattice, runs of 4 096 sweeps. 50 realizations $\{J_{ij}\}$ are considered.

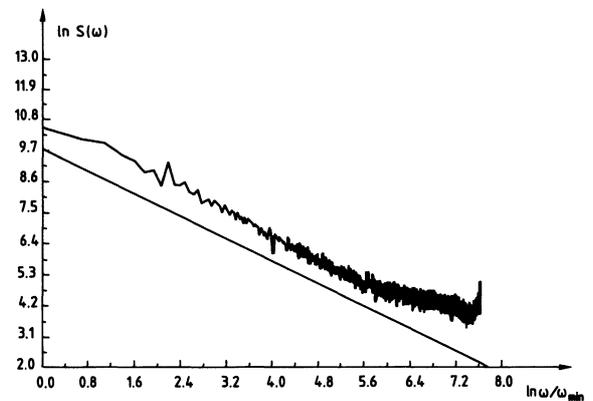


Fig. 2. — $\ln S(\omega)$ (in arbitrary scale) versus $\ln(\omega/\omega_{\min})$, at $T = 1.0$. Same features as in figure 1, but MC dynamics. The straight line is the slope of $S(\omega) \sim 1/\omega$.

A finite small magnetic field h does not affect the behaviour of the spectrum (see Figs. 5a and b) : a slow crossover to a white noise behaviour seems to be present only above $h^* \neq 0$.

Finally, we checked the absence of finite volume effects from our computations : no significant differences appear when going to a 32^2 lattice at the observation time we used. Globally we used the equivalent of ~ 3 hours of CDC 7600.

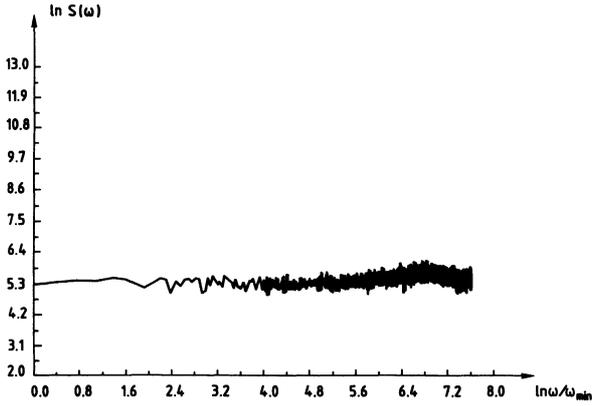


Fig. 3. — Same as in figure 2, but at $T = 2.8$. White noise is evident here.

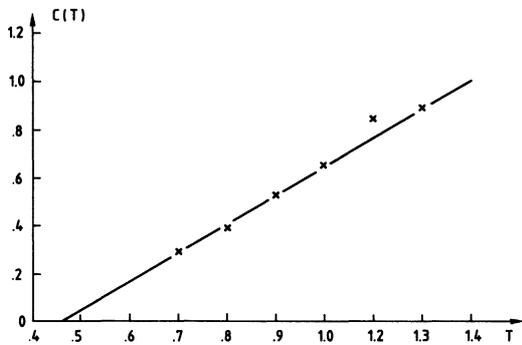


Fig. 4. — $C(T)$ versus T . $S_m(\omega) \sim C(T) \omega^{-1}$. Heat Bath.

4. At the lower critical dimension.

In two dimensions (and less) a continuous symmetry cannot be spontaneously broken. A simple-minded argument runs as follows : consider, for sake of clearness, the Hamiltonian

$$H = \sum_{\langle ik \rangle} \sum_{\alpha=1}^N \sigma_i^\alpha \sigma_k^\alpha, \quad (4.1)$$

where the first sum runs over the nearest neighbours, the index α runs from 1 to N (components of the spin σ), and the spins satisfy the constraint

$$\sum_\alpha (\sigma_i^\alpha)^2 = 1, \quad \forall i. \quad (4.2)$$

H is invariant under $O(N)$ rotations, and the symmetry is spontaneously broken if

$$\langle \sigma_i \rangle \neq 0. \quad (4.3)$$

We can choose the direction of the breaking, let us say

$$\langle \sigma_i^N \rangle \neq 0, \quad \langle \sigma_i^{\alpha \neq N} \rangle = 0. \quad (4.4)$$

In such a situation the correlation of the transverse

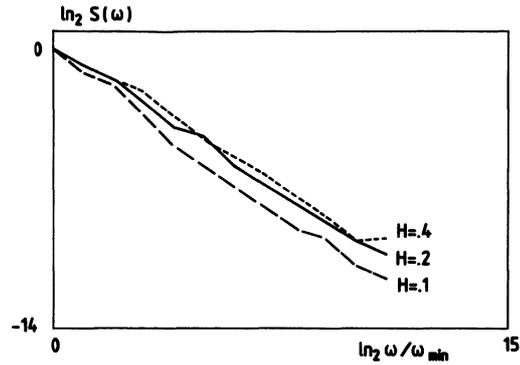


Fig. 5a. — $\ln_2 S(\omega)$ (in arbitrary scale) versus $\ln_2 \omega/\omega_{\min}$ at $T = 0.8$, in presence of $h = 0.1, 0.2, 0.4$. Heat Bath. A fit to $S(\omega) \sim \omega^{-\alpha}$ gives $\alpha = 1$ in the 3 cases.



Fig. 5b. — Same as in 5a, but $T = 1.0$, and $h = 0.1, 0.4, 1.0$. A fit gives $\alpha(h = 0.1) = 1.00$, $\alpha(h = 0.4) = 0.90$, $\alpha(h = 1.0) = 0.69$.

modes is given by

$$G^{\alpha\beta}(p) \propto \delta_{\alpha\beta} \frac{1}{p^2}, \quad \alpha, \beta < N, \quad (4.5)$$

near $p = 0$. This behaviour is clearly inconsistent in two dimensions : in this case we should have ⁽²⁾

$$1 = \langle \sum_\alpha (\sigma_i^\alpha)^2 \rangle > \int d^2p \left(\frac{1}{p^2} + O(1) \right) = \infty. \quad (4.6)$$

The extension of this argument to a more rigorous one is not problematic.

We will argue that in such a situation a $1/\omega$ behaviour has to be seen. It is well known that the low temperature expansion for rotational invariant quantities like

$$G(p) \equiv \int dx e^{ipx} G(x), \quad (4.7)$$

⁽²⁾ We look only at $p \sim 0$: this is the relevant behaviour for our argument.

where

$$G(x) = \sum_{\alpha=1}^N \langle \sigma^\alpha(0) \sigma^\alpha(x) \rangle \quad (4.8)$$

gives correct results. One finds that [15]

$$G(p) = 1 + T(N-1) \frac{1}{p^2}, \quad (4.9)$$

where the Fourier transform of $1/p^2_+$ is given by

$$\int dp (e^{ipx} - 1) \frac{1}{p^2} \sim -\ln x, \quad x \rightarrow \infty. \quad (4.10)$$

We find

$$G(x, T) \sim 1 - T(N-1) \ln x. \quad (4.11)$$

The perturbative expansion breaks down when

$$T(N-1) \ln x = O(1). \quad (4.12)$$

For $N = 2$, x large, it is commonly believed that

$$G(x) \sim x^{-\eta(T)}, \quad \text{for } 0 < T < T_c \quad (4.13)$$

with

$$\eta(0) = 0, \quad \eta(T_c) = \frac{1}{4}. \quad (4.14)$$

For $N > 2$

$$G(x) \sim e^{-m|x|}, \quad (4.15)$$

with

$$m \sim e^{-c/T} \quad \text{when } T \rightarrow 0. \quad (4.16)$$

In the same way when using a dynamical approach [16] one finds that the time dependent correlation functions are given by (at the first order in T)

$$\begin{aligned} G(k, \omega) &= \int d^2x d\tau \sum_{\alpha=1}^N \langle \sigma^\alpha(0, 0) \sigma^\alpha(x, \tau) \rangle e^{i(kx - \omega\tau)} \\ &= \delta(\omega) \delta(k) + \frac{T}{k^4 + \omega^2}. \end{aligned} \quad (4.17)$$

This result implies that if we choose as the interesting observable the total magnetization, its spectrum will behave as ω^{-2} , while if we consider a spin in a given point we will find a $1/\omega$ behaviour. This holds at the first order in T . Taking care of the higher

order corrections it is natural to guess that

$$P_{\sigma_i}(\omega) \sim \frac{1}{\omega^{1-\delta(T)}}, \quad \text{for } N = 2, \quad 0 < T < T_c, \quad (4.18)$$

with $\delta(0) = 0$, and that

$$P_{\sigma_i}(\omega) \sim \frac{1}{\omega} \ln(\omega)^{c(N)}, \quad \text{for } N > 2, \quad (4.19)$$

in the range

$$e^{-c/T} \ll \omega \ll 1. \quad (4.20)$$

We finally remark that in the one-dimensional dynamical Ising model [17] a non trivial behaviour of $P_{\sigma_i}(\omega)$ is found. Namely from the Glauber exact solution one finds

$$P_{\sigma_i}(\omega) \sim \omega^{-1/2} \quad (4.21)$$

at low frequencies.

5. Conclusions.

In this paper we have identified two possible mechanisms for producing $1/f$ noise at equilibrium (on the observation times) : the jumping among different equilibrium states of an amorphous system and the existence of a very slow relaxation time for a system at the lower critical dimension. These results imply that many relatively simple systems display $1/f$ noise.

As far as $1/f$ noise in resistors is concerned, a model for the conductivity should be studied in detail. However it is our belief that $1/f$ noise should be connected to general properties. It would be not surprising if very simple models, like a site dependent random potential, would turn out to display $1/f$ noise at finite temperature, in presence of electron interactions, in two and possibly three dimensions.

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