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SUPERMULTIPLET BREAKING IN NUCLEAR MUON CAPTURE

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Résumé. — Dans l'étude de l'émission de neutrons après capture de muons par un noyau, nous avons montré que la symétrie des supermultiplets est brisée si l'on tient compte de l'interaction dans l'état final entre neutron émis et noyau résiduel. Cette violation de symétrie est due au terme spin-orbite du potentiel optique et dépend de l'énergie du neutron émis. Elle est appréciable pour des neutrons de basse énergie (< 30 MeV) mais devient négligeable pour des neutrons d'énergie plus élevée.

Abstract. — Supermultiplet symmetry is broken in the muon capture by ^{40}Ca with the emission of neutrons when the final state interaction is taken into account. This breaking of symmetry is attributed to the spin-orbit term in the optical potential and is found to be large at lower neutron energies (< 30 MeV). However, we find that high energy neutrons seem to be unaffected by the spin-orbit interaction, and the symmetry is good in this case.

1. **Introduction.** — In the study of total muon capture rates in nuclei, the assumption that all the nuclear matrix elements (not involving the nucleon-momentum) are equal, which is a consequence of the supermultiplet theory of Wigner [1], has often been made. Luyten, Rood and Tolhoek [2] have shown that this assumption holds for double closed shell nuclei if a simple shell model is used to describe the nuclear system. Foldy and Walecka [3] have given a more general proof by using the supermultiplet theory of Wigner [1]. The muon capture matrix elements were calculated in the particle-hole theory taking into account the ground state correlations by random phase approximation, by de Forest [4] and it was found that the symmetry holds good to within 13 %. Since spin-dependent forces are known to be very important in the nuclear interactions, one may doubt the validity of this symmetry, as Wigner's theory assumes a spin-independent nuclear force. Rho [5] has considered this and concludes that the spin-dependent forces do not play a significant role. Walker [6] and Barrett [7] arrived at the same conclusion.

The usefulness of supermultiplet symmetry in muon capture is based upon the recognition that the operators appearing in the vector, axial-vector and pseudo-scalar nuclear matrix elements have the properties of components of a vector operator under the group SU(4) and one can expect the SU(4) symmetry to be broken in the presence of strong spin-orbit interactions. So far, muon capture processes without emission of neutrons have been considered. It is the purpose of this paper to examine the validity of supermultiplet symmetry in muon capture process with the emission of neutrons, first by neglecting the final state interaction and then by taking it into account.

2. **Muon capture nuclear matrix elements.** — The three vector operators relevant to the discussion are

$$J_\alpha = \frac{1}{2} \sum_{n=1}^{A} \tau^\alpha(n)\, t(n)$$

$$Y_\lambda = \frac{1}{2} \sum_{n=1}^{A} \sigma_\lambda(n)\, t(n)$$

$$Y_\lambda^\alpha = \frac{1}{2} \sum_{n=1}^{A} \tau^\alpha(n)\, \sigma_\lambda(n)\, t(n) \tag{1}$$

where τ and σ are isospin and Pauli spin operators for the nucleons with $\alpha, \lambda = 1, 2, 3$ and t is the single nucleon transition operator. For muon capture

$$t(n) = \exp[- i\mathbf{v}_{\rm ab} . \mathbf{r}(n)] \tag{2}$$

where $\mathbf{v}_{\rm ab}$ is the momentum of the neutrino, a and b the initial and final nuclear states and $\mathbf{r}(n)$ the position vector of the nth nucleon. Confining ourselves to

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muon capture by closed shell or subshell nuclei, the nuclear matrix elements are given by

$$M_V^2 = \sum_b \left(\frac{v_{ab}}{m_\mu}\right)^2 |\langle b | O_v | a \rangle|^2$$

$$M_A^2 = \sum_b \left(\frac{v_{ab}}{m_\mu}\right)^2 |\langle b | O_A | a \rangle|^2$$

$$M_P^2 = \sum_b \left(\frac{v_{ab}}{m_\mu}\right)^2 |\langle b | O_p | a \rangle|^2 \quad (3)$$

where m_μ is the mass of the muon and O_v, O_A and O_p are given by eq. (1) with $t(n)$ of eq. (2). It is a valid assumption to replace v_{ab}^2 by $\langle v_{ab}^2 \rangle$ outside the summation over b in eq. (3). Doing this and defining $M_1 = \langle b | O_v | a \rangle$ and $\mathbf{M}_2 = \langle b | O_A | a \rangle$ we can rewrite eq. (3) as

$$M_A^2 = \frac{\langle v_{ab} \rangle^2}{m_\mu^2} \sum_b \mathbf{M}_2 . \mathbf{M}_2^*$$

$$M_v^2 = \frac{\langle v_{ab} \rangle^2}{m_\mu^2} \sum_b M_1 . M_1^*$$

$$M_p^2 = \frac{\langle v_{ab} \rangle^2}{m_\mu^2} \sum_b |\hat{v} . \mathbf{M}_2|^2 . \quad (4)$$

Thus, the supermultiplet symmetry which implies $M_A^2 = M_v^2 = M_p^2$ gives the condition

$$\sum_b M_1 M_1^* = \sum_b \mathbf{M}_2 . \mathbf{M}_2^* = \sum_b |\hat{v} . \mathbf{M}_2|^2 . \quad (5)$$

It is this condition that we wish to examine in muon capture by ^{40}Ca with neutron emission.

3. **A simple shell-model treatment.** — We assume a direct interaction mechanism [8] which is valid for high energy neutrons. In the simple shell model approach [9], the proton that captures the muon is represented by the *j-j* coupled model wave function $| u_{nL}(r) ; L \frac{1}{2} JM \rangle$ where $u_{nL}(r)$ is the radial wave function for the proton and $| L \frac{1}{2} JM \rangle$ is the angular momentum part. This proton makes a transition to the neutron continuum state $| p_n ; \frac{1}{2} m_S \rangle$ of momentum $\mathbf{p}_n$. In the above model, the initial ^{40}Ca and final ^{39}K nuclei wave functions are Slater determinants constructed on single particle wave functions. The nuclear matrix elements are the matrix elements of the one-body operators and are of the form $\langle l_i | O | l_n \rangle$ where l_i represents the occupied state in ^{40}Ca ground state and l_n the emitted neutron state. The sum over final states $| b \rangle$ will imply in this model a sum over the occupied single-particle states i.

So the nuclear matrix elements M_1 and M_2 are

$$M_1 = \left\langle p_n, \frac{1}{2} m_S \right| e^{-i\mathbf{v}.\mathbf{r}} \left| u_{nL}(r) ; L \frac{1}{2} JM \right\rangle (\varphi_\mu)_{av} \quad (6)$$

and

$$\mathbf{M}_2 = \left\langle p_n, \frac{1}{2} m_S \right| \frac{\boldsymbol{\sigma}}{\sqrt{3}} e^{-i\mathbf{v}.\mathbf{r}} \left| u_{nL}(r) ; L \frac{1}{2} JM \right\rangle (\varphi_\mu)_{av} \quad (7)$$

where φ_μ is the muon wave function in 1S atomic orbit which is assumed to be constant over the nuclear volume and $(\varphi_\mu)_{av}$ is its average value. If the outgoing neutron is described by a plane-wave then, for instance, M_2 can be written as :

$$\mathbf{M}_2 = \left\langle \frac{1}{2} m_S \right| e^{i\mathbf{p}_R.\mathbf{r}} \frac{\boldsymbol{\sigma}}{\sqrt{3}} \left| u_{nL}(r) ; L \frac{1}{2} JM \right\rangle (\varphi_\mu)_{av} \quad (8)$$

where

$$\mathbf{p}_R = -(\mathbf{v} + \mathbf{p}_n) .$$

Expanding $e^{i\mathbf{p}_R.\mathbf{r}}$ in spherical waves and writing [10]

$$\left| L \frac{1}{2} JM \right\rangle = \sum_{m_L} C\left(L \frac{1}{2} J ; m_L \, \mu M\right) \left| L m_L \right\rangle \left| \frac{1}{2} \mu \right\rangle .$$

We find that only one angular momentum state, namely $l = L$ yields a non-vanishing contribution [11]. Thus :

$$\mathbf{M}_2 = \sum_{m_L} \left\langle \frac{1}{2} m_S \right| \frac{\boldsymbol{\sigma}}{\sqrt{3}} \left| \frac{1}{2} \mu \right\rangle \times \times 4 \pi (i)^L (-1)^{m_L} Y_L^{-m_L}(\hat{p}_R) \, C\left(L \frac{1}{2} J ; m_L \, \mu M\right) F(nLp_R) (\varphi_\mu)_{av} \quad (9)$$

where

$$F(n, L, p_R) = \int_0^\infty j_L(p_R \, r) \, u_{nL}(r) \, r^2 \, dr .$$

It then follows from angular momentum couplings, for a given closed shell, that,

$$M_A^2 = 4 \pi \frac{\langle v_{ab} \rangle^2}{m_\mu^2} (2J + 1) \, | F(n, L, p_R) |^2 \, | (\varphi_\mu)_{av} |^2 . \quad (10)$$

Similarly one can show that $\sum_b M_1 M_1^*$ and $\sum_b |\hat{v} . \mathbf{M}_2|^2$ assume the same value, if the outgoing neutron is described by a plane wave. The above relations have been derived for a given closed shell. In our model, as mentioned before, the summation over b implies the summation over occupied single particle states and when this summation is carried out, it turns out that the supermultiplet symmetry holdsgood. Thus we see that supermultiplet symmetry is good if the final state interaction is neglected.

4. **Final-state interaction.** — The simple picture described in section 3 is not found to give agreement with the experiments on the asymmetry coefficient of

the angular distribution of the emitted neutron. In particular, Bouyssy and Vinh Mau [12] have shown that the agreement is achieved if the final state interaction is taken into account. In this section we examine the validity of eq. (5) with the neutron-nucleus interaction described by an optical potential of Saxon-Woods [13] form, namely,

$$v(r) = v_R\, f(r) + iv_I\, v(r) + [v^R_{s.o} + iv^I_{s.o}]\, g(r)\, \mathbf{l}.\boldsymbol{\sigma} \quad (11)$$

where v_R, v_I, $v^R_{s.o}$ and $v^I_{s.o}$ are the strengths in MeV of the real central, surface-imaginary, real spin-orbit and imaginary spin-orbit potentials respectively and their numerical values are taken from Van-Oers [14]. The other quantities eq. (11) are given by

$$f(r) = [1 + \exp\{-(r - R_1)/a_1\}]^{-1}$$
$$v(r) = \exp[-\{(r - R_2)/a_2\}^2] \quad (12)$$

and

$$g(r) = \frac{1}{r}\frac{d}{dr} f(r)$$

where R_1, a_1 and R_2 and a_2 are half-fall-off radius and diffuseness parameter for real and imaginary well respectively.

The introduction of the optical potential for the outgoing neutron essentially consists in the use of the solution of Schrödinger equation. The presence of spin-orbit coupling in the optical potential gives the following expression for the final state for the neutron :

$$| f \rangle = 4\pi \sum_{l_f, m_{l_f}, J_f} (i)^{-l_f}\, g_{l_f J_f}(nr)\, (-1)^{m_{l_f}}\, Y_{l_f}^{-m_{l_f}}(\hat{n})$$
$$C(l_f \tfrac{1}{2} J_f\,;\, m_{l_f}, m_{s_f}, M_f)\, | l_f \tfrac{1}{2} J_f\,;\, M_f \rangle \quad (13)$$

where $g_{l_f J_f}(nr)$ is the solution of radial Schrödinger equation with optical potential and the initial proton state

$$| i \rangle = u_{n_i l_i J_i}(r)\, | l_i \tfrac{1}{2} J_i\,;\, M_i \rangle \quad (14)$$

where $u_{n_i l_i J_i}(r)$ is the radial wave function of the bound proton in Saxon-Woods basis. With these initial and final states, the nuclear matrix elements in eq. (5) can be evaluated. The procedure is lengthy and so we give the final results only. For a given closed shell we have :

$$M_1 M_1^* = 64\pi^3 \sum_{l_f, J_f, l} [J_i][J_f](-1)^{J_i - J_f}\, | F_0(l_f\, lnv) |^2$$
$$(\varphi_\mu)^2_{av} \langle l_f \tfrac{1}{2} J_f \| Y_l(\hat{r}) \| l_i \tfrac{1}{2} J_i \rangle \langle l_i \tfrac{1}{2} J_i \| Y_l(\hat{r}) \| l_f \tfrac{1}{2} J_f \rangle \quad (15)$$

$$\mathbf{M}_2.\mathbf{M}_2^* = \frac{64\pi^3}{3} \sum_{l_f J_f l \lambda} (-1)^{J_i - J_f} [J_i][J_f]\, | F_0(l_f\, lnv) |^2 \langle l_f \tfrac{1}{2} J_f \| (Y_l(\hat{r}) \times \sigma_1)_\lambda \| l_i \tfrac{1}{2} j_i \rangle$$
$$\langle l_i \tfrac{1}{2} J_i \| (Y_l(\hat{r}) \times \sigma_1)_\lambda \| l_f \tfrac{1}{2} J_f \rangle (\varphi_\mu)^2_{av} \quad (16)$$

$$| \hat{v}.\mathbf{M}_2 |^2 = \frac{64\pi^3}{3} \sum_{l_f J_f l' \lambda} (-1)^{J_i - J_f} [J_i][J_f][l][l'][\lambda]^{-2}$$
$$(i)^{l' - l}\, C(l\, 1\, \lambda\,;\, 000)\, C(l'\, 1\, \lambda\,;\, 000)\, (\varphi_\mu)^2_{av}\, F_0^*(l_f\, lnv)\, F_0(l_f\, l'\, nv)$$
$$\langle l_f \tfrac{1}{2} J_f \| (Y_l(\hat{r}) \times \sigma_1)_\lambda \| l_i \tfrac{1}{2} J_i \rangle \langle l_i \tfrac{1}{2} J_i \| (Y_{l'}(\hat{r}) \times \sigma_1)_\lambda \| l_f \tfrac{1}{2} J_f \rangle \quad (17)$$

where

$$F_0(l_f\, lnv) = \int_0^\infty u_{n_i l_i J_i}(r)\, j_l(vr)\, g_{l_f J_f}(nr)\, r^2\, dr\,, \qquad [l] = (2\, l + 1)^{1/2},\ (\varphi_\mu)^2_{av}$$

is the square of the average value (over the nuclear volume) of the muon wave function, and the reduced matrix element is defined in reference [15]. The above nuclear matrix elements are evaluated [16], using the optical model parameters of Van-Oers [14].

The numerical values of these nuclear matrix elements are given in table I, after normalizing them with $M_1 M_1^*$. The above nuclear matrix elements are evaluated for each of the contributing closed shells and a summation over the shells is made.

In order to check our calculations we have evaluated the asymmetry coefficient of the angular distribution of the emitted neutron in muon capture by ^{40}Ca. We take into account the momentum dependent terms also and so there will be additional nuclear matrix elements. Our results for the asymmetry coefficient are compared with those of Bouyssy and Vinh Mau [12].

TABLE I

Numerical values of the nuclear matrix elements in eq. (5) *after normalizing by setting* $M_1 M_1^* = 1$

Neutron Energy MeV	M_V^2	M_A^2	M_P^2
10.0	1	0.103 2	0.355 3
20.0	1	0.286 4	0.531 1
30.0	1	0.630 2	0.860 8
40.0	1	0.811 6	1.054 0
50.0	1	0.843 3	1.094 0

5. **Discussion.** — From table I, we find that the condition (5), breaks down in the case of muon capture with neutron emission, when the interaction of the outgoing neutron with the residual nucleus is taken into account through an optical potential. This is due to the spin-orbit term in the optical potential, as the condition (5) assumes no spin-dependent nuclear forces. The absence of such a term would reduce the formalism of section 4 to that of section 3 with a modification of the neutron momentum and then the condition (5) would be satisfied. Further, as the energy of the neutron is increased beyond 30 MeV, the condition (5) seems to hold. This could be due to the fact that high energy neutrons (> 30 MeV) are almost unaffected by the spin-orbit interaction. A similar situation is shown to exist in the studies of Bouyssy, Ngo and Vinh Mau [17]. When the curve 2 in figure 2 of the above reference (which does not include spin-orbit term) is compared with curve 1 of figure 2 (which includes spin-orbit term), it is found that the asymmetry coefficient for higher neutron energies (> 50 MeV) is unaffected by the spin-orbit interaction.

TABLE II

Asymmetry coefficient of the angular distribution of neutrons in muon capture by ^{40}Ca *after including the final state interaction* (FSI) *through the optical potential. The last column corresponds to reference* [12].

Neutron Energy MeV	No FSI	With FSI	Ref. [12] Values
10	− 0.110	+ 0.126 8	− 0.012
20	− 0.067	+ 0.149 8	+ 0.144
30	− 0.022	+ 0.039 6	+ 0.139
40	+ 0.023	+ 0.072 7	+ 0.063
50	+ 0.068	+ 0.101 0	+ 0.096

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